irrational-valued dimensionless noncommutativity parameters, the rank of the gauge group of the regulating commutative theory must be taken to infinity as one takes the continuum limit. As a special case, the construction includes the recent proposal of using twisted large N reduced models [11] for a concrete definition of noncommutative gauge theory. We shall find, however, that the class of noncommutative Yang-Mills theories that one can obtain using twisted large N reduced models is not the most general one.

The formalism also allows for the coupling of gauge fields to matter fields in the fundamental representation of the gauge group. We will carry out a large mass expansion of the matter-coupled theory and clarify the physical meaning of Wilson loops in noncommutative gauge theories [15, 22, 24, 34, 35]. A remarkable property of these Wilson loops [15] is that star-gauge invariance does not require that the loop be closed, but it does require that the relative distance vector between the two ends of the loop be proportional to its total momentum. Namely, as one increases the total momentum, the loop extends in spacetime. This is another manifestation of UV/IR mixing due to the noncommutativity. We will demonstrate how noncommutative Wilson loops naturally arise from matter field averages in noncommutative gauge theory. By calculating the effective action induced by matter and star-gauge invariant observables constructed out of the matter fields, we shall see that noncommutative Wilson loops play a very fundamental role, just like ordinary Wilson loops do in commutative gauge theories [29]. We will also show explicitly how star-gauge invariant observables reduce smoothly to ordinary gauge invariant observables in the commutative limit for fixed gauge backgrounds.

Morita equivalence in the presence of fundamental matter fields is also proven. As a special case, we obtain the twisted Eguchi-Kawai model with fundamental matter which was introduced in Ref. [36] as a model which reproduces ordinary large N gauge theory in the Veneziano limit. These Morita equivalences clarify the interpretations of various quantities in noncommutative gauge theory. For instance, it is known that only planar noncommutative Feynman diagrams contribute to the beta-function at one-loop order [14, 16, 37], and in the case of noncommutative quantum electrodynamics it is given by [37]

$$\beta(g^2) = -\frac{g^4}{4\pi^2}\left(\frac{11}{3} - \frac{2}{3}n_f\right) , \tag{1.1}$$

where n_f is the number of fermion flavours. The beta-function (1.1) coincides with that of ordinary U(N) Yang-Mills theory coupled to $N_f = n_f N$ flavours of fermion fields in the large N (planar) limit (after an appropriate rescaling of the Yang-Mills coupling constant g). This coincidence can be understood as a consequence of the Morita equivalence that we shall derive. In fact, the results of such calculations indicate that the phenomenon of Morita equivalence in noncommutative gauge theories holds beyond the classical level in regularized

perturbation theory. We will further show that the star-gauge invariant observables of two Morita equivalent noncommutative Yang-Mills theories are in a one-to-one correspondence. Owing to this fact, we can define regularized correlation functions of star-gauge invariant observables in noncommutative Yang-Mills theory with multi-valued gauge fields by using the lattice regularization of the corresponding Morita equivalent noncommutative gauge theory with periodic gauge fields. This Morita equivalence property will be the key feature which permits the discretization of generic noncommutative Yang-Mills theories in the continuum.

The organization of the remainder of this paper is as follows. In Section 2 we present a pedagogical introduction to noncommutative field theory, and in particular noncommutative Yang-Mills theory, for the sake of completeness and in order to set up the notations to be used throughout the paper. In Section 3, we present a detailed and very general field theoretical derivation of the Morita equivalence relation between noncommutative gauge theories and demonstrate the one-to-one correspondence between star-gauge invariant observables in the Morita equivalent theories. In Section 4, we construct the discrete version of noncommutative field theory. We show that the lattice formulation exhausts the noncommutative Yang-Mills theories in arbitrary even dimensions, given the Morita equivalence properties described in Section 3. In Section 5 we establish Morita equivalence on the lattice, which allows us to regularize noncommutative Yang-Mills theories by means of commutative lattice gauge theories with twisted boundary conditions on the gauge fields. We also describe the relationship to finite dimensional matrix model representations of noncommutative gauge theories. Finally, in Section 6 we introduce matter fields in the fundamental representation of the gauge group and study the properties of star-gauge invariant observables as well as Morita equivalence in the presence of matter fields.

2 Quantum field theory on noncommutative spaces

In this Section we will briefly review some aspects of constructing noncommutative gauge theories in the continuum, in a way that will enable us to later construct a lattice version of the field theory. We will start with a simple scalar field theory to introduce the ideas, and then move on to a description of noncommutative Yang-Mills theory and its observables.

2.1 Scalar field theory

Consider the scalar quantum field theory which is described by the partition function

$$\begin{aligned} Z &= \int \mathcal{D}\phi \; \mathrm{e}^{-S[\phi]} \\ S &= \int \mathrm{d}^D x \left(\frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{1}{2}\phi^2 + \frac{1}{4!}\phi^4\right) , \end{aligned} \tag{2.1}$$